Azimuthal decorrelation of forward and backward jets at the Tevatron

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Abstract

We analyse the azimuthal decorrelation of 'Mueller-Navelet' dijets produced in the $p\bar{p}$ collisions at Tevatron energies using a BFKL framework which incorporates dominant subleading effects. We show that these effects significantly reduce the decorrelation yet they are still insufficient to give satisfactory description of experimental data. However a good description of the data is obtained after incorporating within formalism the effective rapidity defined by Del Duca and Schmidt.

An ideal test of $\log(1/x)$ dynamics is the "Mueller-Navelet" process [1, 2, 3] in hadron-hadron collisions with one backward jet separated from a forward jet by a large rapidity interval. The test has been under study for several years, but has recently been sharpened by the considerable improvement in the precision of the data [4]. The relevant hard scale is specified by the transverse momenta of the jets $(\boldsymbol{k}_1, \boldsymbol{k}_2)$. An advantage of this process is that these momenta can be of comparable magnitude so the intervening DGLAP evolution is suppressed. The behaviour of the cross-section is therefore sensitive to the diffusion of transverse momenta of the intermediate gluons, which is a property of leading $\log(1/x)$ dynamics [5, 6]. The characteristic features of this process are (i) a rapid increase of the 2-jet production cross section with increasing incident energy and (ii) a weakening of the azimuthal back-to-back correlation between the jets as the rapidity interval is increased. The analyses have so far been within the leading order $\log(1/x)$ BFKL framework, and indicate much more decorrelation than shown by the data. The improvement of the 2-jet data has been accompanied by progress in understanding and quantifying the effect of sub-leading $\log(1/x)$ corrections. The full NLO BFKL $(\log(1/x))$ contributions have been obtained and found to be large [7, 8, 9]. However the major contribution can be resummed to all orders in a straightforward way [10, 11, 12]. To be precise the resummation is effected by requiring that the virtuality of the intermediate gluons is dominated by their transverse momenta squared, which although formally sub-leading, is implicit in the derivation of the BFKL equation. We call this the consistency condition (CC). The resummation is found to stabilise the solution of the NLO BFKL equation. The purpose of this paper is to incorporate those subleading effects generated by the consistency constraint in the theoretical description of the dijet production in hadronic collisions.

The cross-section describing the production of a forward jet (x_1, k_1^2) and a backward jet (x_2, k_2^2), with relative azimuthal angle $\phi - \pi$, may be written

$$\frac{d\sigma}{dx_1 dx_2 dk_1^2 dk_2^2 d\phi} = \frac{1}{x_1 x_2} \left[x_1 f_{\text{eff}}(x_1, k_1^2) \, x_2 f_{\text{eff}}(x_2, k_2^2) \right] \frac{d\hat{\sigma}}{dk_1^2 dk_2^2 d\phi}, \tag{1}$$

where $f_{\text{eff}} = g + \frac{4}{9}(q + \bar{q})$ is the effective parton distribution of the incoming proton (antiproton). Note, that with this convention the back-to-back configuration of the jets in the transverse plain corresponds to $\phi = 0$. The hard partonic cross section (defined here for the gluon-gluon scattering) is of the form

$$\frac{d\hat{\sigma}}{dk_1^2 dk_2^2 d\phi} = \frac{\bar{\alpha}_s^2 \pi}{4(k_1 k_2)^3} \sum_{m=0}^{\infty} \cos(m\phi) I_m(\Delta Y, \rho) \tag{2}$$

where $\bar{\alpha}_s = 3\alpha_s/\pi$,

$$\Delta Y = \log\left(\frac{x_1 x_2 s}{k_1 k_2}\right), \quad \rho = \log\left(\frac{k_1^2}{k_2^2}\right), \tag{3}$$

and I_m is the solution of the BFKL equation with the consistency condition imposed. In this paper we shall use a fixed strong coupling constant corresponding to $\bar{\alpha}_s = 0.15$. The solution may be written

$$I_m(\Delta Y, \rho) = \int_0^{\infty} d\nu \int \frac{d\omega}{2\pi i} \, \frac{1}{\omega - \chi_m(\omega, \nu)} e^{\omega \Delta Y} \cos(\rho \nu) \tag{4}$$

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where the kernels χ_m are given by[1]

$$\chi_m(\omega, \nu) = \bar{\alpha}_s \left[2\psi(1) - \psi \left(\frac{m + \omega + 1}{2} + i\nu \right) - \psi \left(\frac{m + \omega + 1}{2} - i\nu \right) \right] \tag{5}$$

with ψ as usual, the logarithmic derivative of the Euler Gamma function.

We evaluate the $d\omega$ integral in (4) by picking up the leading singularity $\omega_m^0(\nu)$ which is the solution of the implicit equation

$$\omega_m^0(\nu) = \chi_m(\omega_m^0(\nu), \nu), \tag{6}$$

(cf.(5)), to obtain

$$I_m(\Delta Y, \rho) = \int_0^\infty d\nu R_m^0(\nu) \exp(\omega_m^0(\nu)\Delta Y) \cos(\rho\nu) \tag{7}$$

where the residue R is

$$R_m^0(\nu) = \left[1 - \frac{d\chi_m(\omega, \nu)}{d\omega} \Big|_{\omega=\omega_m^0(\nu)} \right]^{-1}. \tag{8}$$

Hence at LO the above solution (6) would simplify to

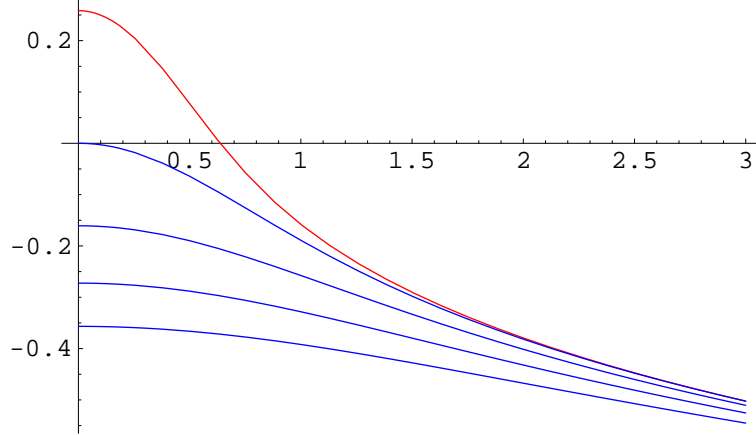$$\omega_m^0(\nu) = \chi_m(0, \nu), \tag{9}$$

with $R_m^0 = 1$.

Figure 1: $\omega_m^0(\nu)$, for $0 \le m \le 4$. In decreasing order, the curves correspond to $m = 0, 1, \dots 4$.

In Fig. 1 we illustrate the solution $\omega_m^0(\nu)$, (6), for various values of m. For $\Delta Y \gg 1$ the dominant contribution comes from the region $\nu \sim 0$. If we then expand around the saddle point, $\nu = 0$, we see that

$$I_m(\Delta Y, \rho) \sim \exp(\omega_m^0(0)\Delta Y). \tag{10}$$

[1]An alternative resummation of subleading $\log(1/x)$ effects, based on keeping the collinearly enhanced contributions [13] is found to give the same result if $\psi(z)$ of (5) is approximated by $1/z$, apart from the inclusion in [13] of running α_s and non-singular terms in the splitting function.

The total forward+backward jet cross section is controlled by the $m = 0$ contribution and the intercept $\omega_0^0(0)$ gives the rate of growth as the rapidity interval increases. We emphasize that the "subleading" intercept $\omega_0^0(0)$ is significantly smaller than its LO counterpart, and significantly above the NLO approximation.

It can also be seen from Fig. 1 that the intercepts $\omega_m^0(0)$ decrease with increasing m. This implies the dominance of the $m = 0$ term at large values of ΔY, which determines the rate of decorrelation of the azimuthal angle between the forward and backward jets. In practice we do not restrict ourselves to the saddle point approximation, but rather evaluate (7) numerically. In Fig. 2 we plot the azimuthal distributions $d\sigma/d\phi$

$$\frac{d\sigma}{d\phi} = \int_\Delta dx_1 dx_2 dk_1^2 dk_2^2 \frac{d\sigma}{dx_1 dx_2 dk_1^2 dk_2^2} \tag{11}$$

where the integration region Δ is restricted by the experimental cuts [4]. To be precise the cuts which we impose are as follows:

(i) the tagged jet tranverse momenta: $|\mathbf{k}_i| > 20$ GeV, and $\max(|\mathbf{k}_1|, |\mathbf{k}_2|) > 50$ GeV;

(ii) the tagged jet pseudorapidities $|\eta_i| < 3.0$ and the pseudorapidity distance between jets $|\Delta\eta = |\eta_2 - \eta_1|$ was required to fall into one of two windows: $2.5 < \Delta\eta < 3.5$ or $4.5 < \Delta\eta < 5.5$.

Fig. 2 shows the results corresponding both to the LO BFKL equation and those which are obtained from the BFKL equation with the subleading effects generated by the consistency constraint. We confront our predictions with the experimental data obtained at the Tevatron. We see from these figures that the subleading effects reduce azimuthal decorrelation obtained from the LO BFKL equation. The resulting ϕ distributions are however still too flat when compared with the experimental data.

It has been pointed out in ref. [15] that the approximations which are adopted within the BFKL formalism may not be accurate, particularly in the region away from the back-to-back configuration of the two jets. It has been proposed that in order to improve accuracy of the predictions one should use the effective rapidity difference $\hat{y}(m)$ for each azimuthal projection m in the place of ΔY in the solutions I_m of the BFKL equation(s). The effective rapidity intervals $\hat{y}(m)$ are defined as:

$$\hat{y}(m) = \Delta Y \frac{\int d\phi \cos(m\phi)(d\sigma/dy_1 dy_2 dk_1 dk_2 d\phi)}{\int d\phi \cos(m\phi)(d\sigma_0/dy_1 dy_2 dk_1 dk_2 d\phi)} \tag{12}$$

where $d\sigma/dy_1 dy_2 dk_1 dk_2 d\phi$ and $d\sigma_0/dy_1 dy_2 dk_1 dk_2 d\phi$ denote the exact $O(\alpha_s^3)$ contribution to the three jet cross section and its BFKL (i.e. large ΔY) approximation respectively.

Guided by this proposal we have used as the asymptotic variable the effective rapidity $\hat{y}(\Delta Y, k_1^2, k_2^2, \phi)$

$$\hat{y}(\Delta Y, k_1^2, k_2^2, \phi) = \Delta Y \frac{d\sigma/(dy_1 dy_2 dk_1 dk_2 d\phi)}{d\sigma_0/(dy_1 dy_2 dk_1 dk_2 d\phi)} \tag{13}$$
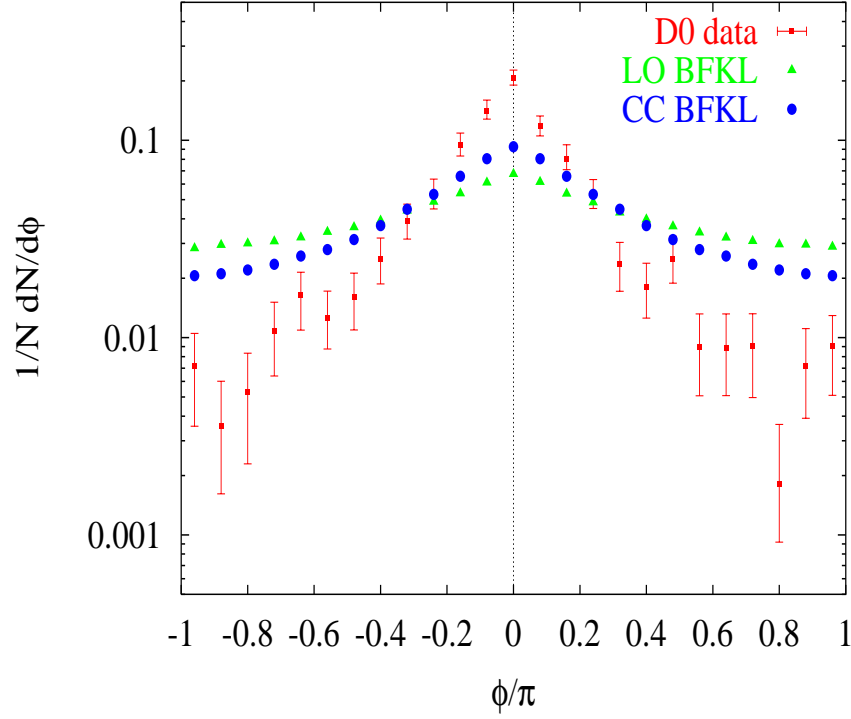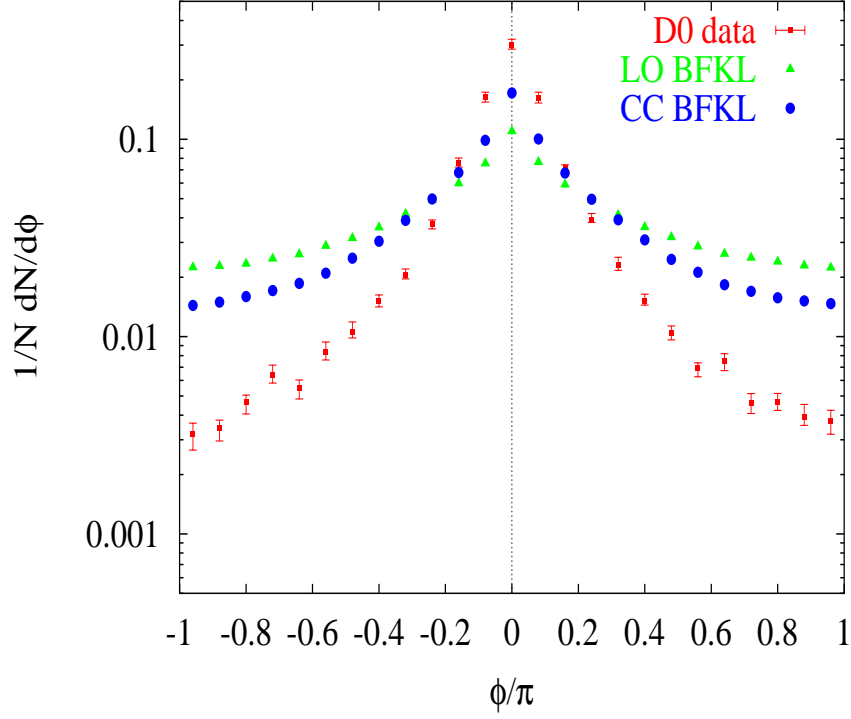
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Figure 2: Azimuthal decorrelation of the forward and backward jets for two rapidity separation intervals: (a) $2.5 < \Delta\eta < 3.5$, (b) $4.5 < \Delta\eta < 5.5$. The D0 data [4] are shown by points with errorbars. The BFKL (LO) and BFKL (CC) predictions are shown by triangles and circles respectively.

where $d\sigma_0/(dy_1 dy_2 dk_1 dk_2 d\phi)$ should now contain the subleading BFKL effects. They just correspond to the replacement:

$$\Delta Y \to \Delta Y - \frac{1}{2} \left| \log \left(\frac{k_1^2}{k_2^2} \right) \right| \tag{14}$$

in the LO BFKL expression for $d\sigma_0/(dy_1 dy_2 dk_1 dk_2 d\phi)$.

The use of effective rapidity $\hat{y}(\Delta Y, k_1^2, k_2^2, \phi)$ defined by equation (13) aims at correcting the large ΔY approximation of the $2 \to 3$ jets cross-section integrated over the rapidity of the third jet. In this approximation the integrated $2 \to 3$ jets cross-section is just proportional to ΔY. This is the result of the fact that the rapidity y_3 of the third jet varies between the rapidities y_1 and y_2 of the tagged jets ($\Delta Y = y_2 - y_1$) and that, in the large ΔY approximation, potential dependence of the momenta of the colliding partons on y_3 is neglected. The analysis performed in ref. [15] shows that this is an inaccurate approximation and that the use of the exact kinematics introduces a large suppression. The effective rapidity $\hat{y}(\Delta Y, k_1^2, k_2^2, \phi)$ incorporates this suppression and reflects the effective rapidity range spanned by the (mini)jets radiated with rapidities between y_1 and y_2 [15]. In the formulae defining the exact 3-jet cross section a fixed coupling was used corresponding to the value of $\bar{\alpha}_s$ which entered the BFKL kernels. Let us mention, that there is a significant dependence of $\hat{y}(\Delta Y, k_1^2, k_2^2, \phi)$ on the ϕ variable. The conditional maximum with other variables fixed is always reached for $\phi = 0$ and if $k_1^2 \simeq k_2^2$ then $\hat{y}(\Delta Y, k_1^2, k_2^2, \phi = 0) \simeq \Delta Y$. We have noticed that the numerical convergence of the integral in (7) is poor if $\hat{y}$ is below 1.2. In this region we have therefore used the perturbative expansion of the solution of the (modified) BFKL equation up to $O(\alpha_s^3)$. This required a careful treatment of the 3 jet cross section for the third jet with low transverse momentum k_3. Integration over the third jet phase space leads to infrared divergence which is cancelled by the virtual correction to the Born term, when integrated over the region of phase space containing the singular point $k_3 = 0$.

The results of the calculations incorporating the effective rapidity are summarised in Fig. 3. These plots show the ϕ distributions which are obtained from the BFKL framework with the subleading effects taken into account and compare with effective rapidity $\hat{y}(\Delta Y, k_1^2, k_2^2, \phi)$ in the place of ΔY as an asymptotic variable. Again we compare with the available experimental data. We see that the use of the effective rapidity significantly improves agreement with the data. In Figures 3(a,b) we also show results obtained from the LO BFKL equation using the corresponding effective rapidity. In contrast to the results of the BFKL calculations presented in Figures 2 (a,b), when the effective rapidity is used the azimuthal decorrelation is far from being saturated (which would correspond to the distribution flat in ϕ) and it should be more sensitive to the particular form of the BFKL kernel. Thus, it is surprising that these LO BFKL predictions do not differ appreciably from those obtained using the BFKL formalism with subleading effects *and* the effective rapidity. This implies that the implementation of effective rapidity suppresses the potentially large effect of subleading corrections. It is mainly caused by the fact that the absolute values of the effective rapidities at the tails of the distributions are rather small (i.e. $\hat{y} \sim 1$), where the cross-section is dominated by the 2 jet + 3 jet production. In fact, the difference which is visible at the tails of the distributions obtained from the BFKL LO and BFKL CC equations does not come from the difference in the cross sections at the tails but rather from the region of central ϕ which influences the tails through entering the common normalisation factor N. Finally, we have checked, that the sensitivity of the results to variations
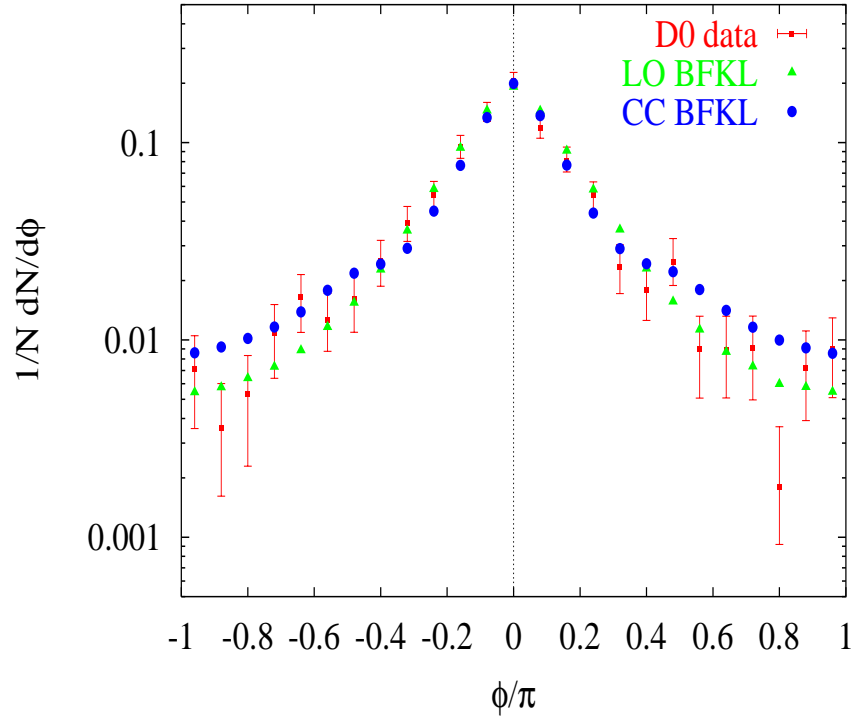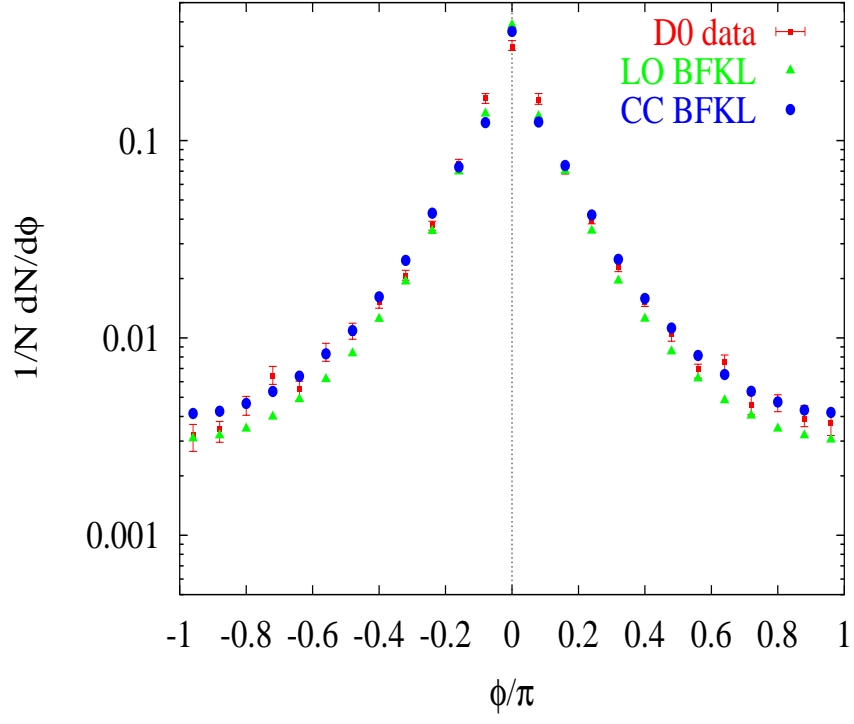
a)

b)

Figure 3: As for Fig. 2 but with the effective rapidity $\hat{y}$ of (13) incorporated in the BFKL predictions.
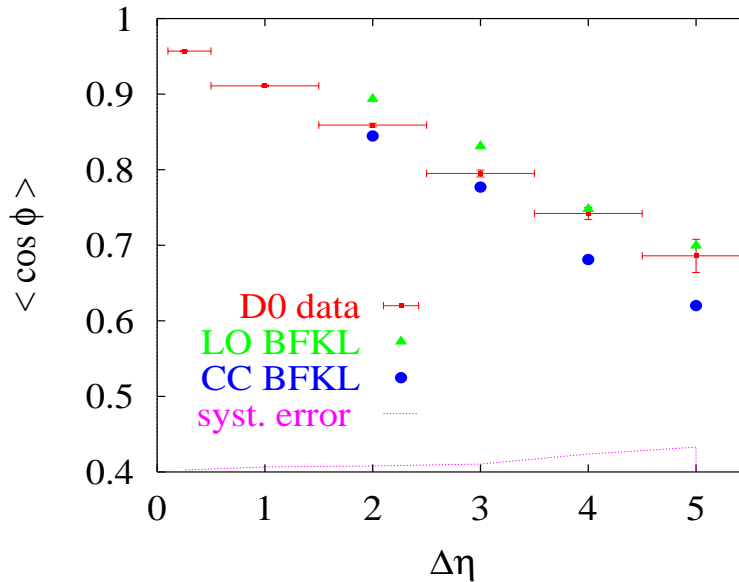
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Figure 4: The average value of $\cos\phi$ for various $\Delta\eta$ intervals. The D0 data (points with errorbars) compared with the BFKL LO calculations (triangles) and with the BFKL CC calculations (circles). The band between the $\Delta\eta$ axis and the dotted line represents the correlated systematic error of the data.

of the fixed $\bar{\alpha}_s$ between 0.13 and 0.18 is small and $\bar{\alpha}_s = 0.15$ gives the best description of the data.

With experimental cuts used in the D0 analysis the cross-section for cental bins of ϕ contains contributions from configurations with $\boldsymbol{k}_1 + \boldsymbol{k}_2 = 0$. In order to describe properly these configurations the full BFKL resummation is necessary. This is due to the fact that the kinematical suppression of additional gluon emissions is weak in this case, and the corresponding effective rapidity $\hat{y}$ is close to ΔY. Thus, $\alpha_s\hat{y}$ becomes large and it is not enough to retain only the first terms of the perturbative expansion. In other words, the $O(\alpha_s^3)$ cross-section, calculated as the sum of the LO 3 jet and NLO 2 jet cross-section, is expected to be subject to a series of large $(\alpha_s\hat{y})^n$ corrections. In fact, we have checked explicitly that without the resummation, the data in the central ϕ bins are not well reproduced.

We also studied the distribution of $\langle\cos\phi\rangle$ in the bins of pseudorapidity separation of the jets $\Delta\eta$. Here we used only the method and parameters which gave the best description of the azimuthal angle distributions of the Mueller-Navelet jets. The results of our calculation are compared to the experimental data [4] in Fig. 4. Both the LO BFKL and CC BFKL equations with the effective rapidity $\hat{y}$ give a resonable description of the experimental data, when the statistical and the correlated systematical errors of the data are taken into account. However, it is clear that some small discrepancy between the theoretical and experimental values still remains.

To summarise we emphasize the following results of our analysis:

(i) The BFKL formalism in its original form fails to describe the data on the azimuthal decorrelation of dijets. The dominant subleading effects do improve the results obtained within the leading logarithmic approximation, but they still generate more decorrelation in the tails of the azimuthal distribution than that required by the data.

(ii) Most of the relevant contribution to the decorrelation in the Tevatron domain is present already in the fixed order $O(\alpha_s^3)$ expressions.

(iii) In order to describe the data it is essential to use the effective rapidity $\hat{y}$ defined by eq. (13) and to use the 2 jet (LO + NLO) + 3 jet cross-section in the domain of low values of $\hat{y}$. In this region the BFKL formalism does not therefore introduce any new information, which would not be present in the fixed order calculation.

(iv) The only places where the use of the complete BFKL framework seems to be necessary are in the very central bins of ϕ.

(v) We therefore conclude that it is possible to describe the decorrelation data using the fixed order perturbative expressions in the tails of the azimuthal distribution and the complete BFKL resummation for $\phi \approx 0$.

Acknowledgements

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